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The Unitary Representation Operators

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Abstract

A representation (called the U-representation) which remains unitary for all spins and for all ranges of velocities was obtained by us in a recent paper. We obtain here relevant expressions for the boosts operator and the observables in such a representation.

1. *Introduction*

Relativistic wave equations in the Schrödinger form, involving a $2(2s + 1)$ component wave function to describe particles and anti-particles of spin s, have received considerable attention in recent years (Weaver *et al.*, 1964; Mathews, 1966b; Mathews, 1967). The wave function is required to transform according to the $D(0, s) + D(s, 0)$ representation of the homogeneous Lorentz group. Since the wave function has just the minimum number of components, no auxiliary conditions are needed. A remarkable and interesting feature of this formalism is that the requirement of manifest covariance, which for many years played a dominant role in the investigation of higher spins, was completely abandoned here. It was Foldy who first suggested that the manifest covariance is really a luxury and the relativistic invariance of a wave equation can be ensured by requiring the solutions of the wave equation to provide a representation space for the generators of the Poincaré group. The wave function which one employs to provide such a representation is not unique, but has many forms, all interrelated by similarity transformations. Accordingly we have, broadly speaking, three different representations, called the ψ-representation (Mathews, 1966a), the Foldy & Wouthuysen (1950, 1966) representation and the E-representation (Cini & Touschek, 1958; Alagar Ramanujam, 1973). Except for the Foldy-Wouthuysen representation, which is applicable to particles with velocities in the low momentum limit, the other two representations, as one can see, are not unitary for spins greater than $\frac{1}{2}$. In a recent paper (Alagar Ramanujam, 1974) a unitary representation (U-

representation) which remains unitary for all spins and for all ranges of velocity was obtained. The advantage of having a unitary representation is that in such a representation the Lorentz invariant scalar product takes the usual simple form and the expressions for momentum and charge density become much simpler when we go the q-number theory. In this paper we complete the work initiated by Alagar Ramanujam (1974) by determining relevant expressions for the boosts operator and the absorvables in the U-representation.

2. *The Unitary Representation*

We give below, for the sake of easy reference, the salient features of the U-representation. Assuming the energy sign operator (H_R/E_R) of a particle in its rest frame to be ρ_1 the corresponding operator (H_u/E) in the U-representation is obtained by a similarity transformation of the form

$$H_u/E = F^{-1}\rho_1 F \tag{2.1}$$

where

$$F = \exp\{\rho_1\lambda_p\theta\} = \sum_{0 \text{ or } \frac{1}{2}}^{s} B_\alpha \cos\alpha\theta + \rho_1 \sum_{0 \text{ or } \frac{1}{2}}^{s} C_\alpha \sin\alpha\theta \tag{2.2}$$

$$\cos\alpha\theta = \frac{\cosh\alpha w}{\sqrt{(\cosh 2\alpha w)}}, \qquad \sin\alpha\theta = \frac{\sinh\alpha w}{\sqrt{(\cosh 2\alpha w)}}; \qquad \tanh w = p/E$$

$$F^{-1} = F^{\dagger} = \exp -(\rho_1\lambda_p\theta) = \Sigma\cos\alpha\theta B_\alpha - \rho_1\Sigma\sin\alpha\theta C_\alpha$$

The U-representation Hamiltonian (H_u) obtained from (2.1) is of the form

$$H_u = \{\exp(-2\rho_1\lambda_p\theta)\}\rho_1 E = \sum_{0 \text{ or } \frac{1}{2}}^{s} \{E\tanh 2\alpha w C_\alpha + \rho_1 E \operatorname{sech} 2\alpha w B_\alpha\} \tag{2.3}$$

$$\boldsymbol{\lambda} = \begin{pmatrix} \mathrm{s} & 0 \\ 0 & -\mathrm{s} \end{pmatrix} = \rho_3\mathbf{S}; \qquad \mathbf{S} = \begin{pmatrix} \mathrm{s} & 0 \\ 0 & \mathrm{s} \end{pmatrix}$$

Here, $\mathrm{s}(s_1, s_2, s_3)$ is a $(2s + 1)$ dimensional spin matrix and ρ_1, ρ_3 are Pauli's first and third matrices, whose elements are taken as $(2s + 1)$ dimensional matrices rather than just numbers. B_α and C_α are even and odd combinations of the projection operator Λ_α to the eigenvalue α of $\lambda_p = (\boldsymbol{\lambda}\cdot\mathbf{p})/p$

$$B_\alpha B_\beta = C_\alpha C_\beta = \delta_{\alpha\beta} B_\alpha \tag{2.4}$$

$$C_\alpha B_\beta = \delta_{\alpha\beta} C_\alpha, \qquad \Sigma B_\alpha = I \tag{2.4a}$$

An operator F links the rest frame wave function ψ_R and the U-representation wave function ψ_u by the form $\psi_R = F\psi_u$. It may also be noted that the wave function ψ_u and the ψ-representation (also called the D-representation) wave function ψ are linked by the form $\psi = A\psi_u$, where

$$A = R^{-1}F = \{\Sigma \overset{(+)}{d_\alpha} B_\alpha + \rho_1 \Sigma \overset{(-)}{d_\alpha} C_\alpha\}F \tag{2.5}$$

$$\overset{(\pm)}{d\alpha} = \pm(1/2)\sqrt{(m/E)}m^{-\alpha}(E+p)^{-\alpha}\{(E+p)^{2\alpha} \pm m^{2\alpha}\}$$

The U-Representation Operators

For every infinitesimal generator G_ψ of the Poincaré group acting on ψ, there is a corresponding generator G_u acting on ψ_u, the latter being related to the former by a similarity transformation of the form $G_u = A^{-1}G_\psi A$. The operator $\mathbf{P} = \mathbf{p} = -i\nabla$ generating translation, and the operator $\mathbf{J} = (\mathbf{x} \times \mathbf{p}) + \mathbf{S}$ generating rotation, commute with A and, as a result, they undergo no change when we go from the ψ-representation to the U-representation. But the case is not so simple when we come to the boosts operator $\mathbf{K} = -i\partial/\partial v$, which generates the Lorentz transformation. To obtain $\mathbf{K}_u$ one has to actually evaluate the form

$$\mathbf{K}_u = A^{-1}\mathbf{K}_\psi A \tag{2.6}$$

with

$$\mathbf{K}_\psi = t\mathbf{p} - \mathbf{x}H_\psi + i\boldsymbol{\lambda}$$

where $H_\psi = H_u = A^{-1}H_\psi A$.

A direct evaluation of the right-hand side of equation (2.6) becomes very involved and tedious. We therefore give below a roundabout but less tedious method. In this connection it may be noted that any vector $\mathbf{B}$ can be expressed in the form

$$\mathbf{B} = [(\mathbf{B}\cdot\mathbf{p})\mathbf{p} - (\mathbf{B} \times \mathbf{p}) \times \mathbf{p}]/p^2 \tag{2.7}$$

To obtain $\mathbf{K}_u \cdot \mathbf{p}$, we take the dot product of the relation given in equation (2.6) and by using the commutation relations like (Mathews, 1966a)

$$-i\mathbf{p}\cdot[\mathbf{x}, \Sigma k_\alpha C_\alpha] = \Sigma p \frac{dk_\alpha}{dp} C_\alpha$$

$$-i\mathbf{p}\cdot[\mathbf{x}, \Sigma b_\alpha B_\alpha] = \Sigma p \frac{db_\alpha}{dp} B_\alpha$$

we obtain

$$(\mathbf{K}_u \cdot \mathbf{p}) = tp^2 - (\mathbf{x}\cdot\mathbf{p})H_u + \frac{i(\boldsymbol{\lambda}\cdot\mathbf{p})(\rho_1 H_u)^2}{2E^2} + \frac{i(\boldsymbol{\lambda}\cdot\mathbf{p})}{2} + \frac{ip^2 H_u}{2E^2} \tag{2.8}$$

To obtain $(\mathbf{K}_u \times \mathbf{p}) \times \mathbf{p}$, we make use of the relation (Mathews, 1966b)

$$\boldsymbol{\Sigma} = \mathbf{J}\frac{E}{m} - \frac{(\mathbf{J}\cdot\mathbf{p})\mathbf{p}}{m(E+m)} + \frac{(\mathbf{K}_\psi \times \mathbf{p})H_\psi}{Em} \tag{2.9}$$

$\boldsymbol{\Sigma}$ is the spin operator in the ψ-representation. By post-multiplying relation (2.9) by H_ψ, taking the cross-product of both sides by $\mathbf{p}$ and making a similarity transformation with A, we get

$$(\mathbf{K}_u \times \mathbf{p}) \times \mathbf{p} = -\{\overset{(+)}{a}_\alpha \rho_1 B_\alpha + \overset{(+)}{d}_\alpha C_\alpha\}\rho_3 p^3 \mathbf{c} - (\mathbf{J} \times \mathbf{p}) H_u + \{\overset{(-)}{d}_\alpha B_\alpha - \overset{(-)}{a}_\alpha \rho_1 C_\alpha\} i p^2 \mathbf{c} \times \mathbf{p} \tag{2.10}$$

$$\overset{(\pm)}{a}_\alpha = -(E/2p^2\sqrt{(\cosh 2w)})[\sqrt{(\operatorname{sech} 2(\alpha - 1)w)} \pm \sqrt{(\operatorname{sech} 2(\alpha + 1)w)}]$$

$$\overset{(\pm)}{d}_\alpha = \pm \sinh 2\alpha w \overset{(\pm)}{a}_\alpha \mp \cosh 2\alpha w \,.\, \tanh w \overset{(\mp)}{a}_\alpha ; \qquad \mathbf{c} = \frac{\boldsymbol{\lambda} \times \mathbf{p}}{p}$$

Combining (2.10), (2.8) and (2.7) we get

$$\mathbf{K}_u = t\mathbf{p} - \mathbf{x}H_u + \frac{i\mathbf{p}\lambda_p(\rho_1 H_u)^2}{2pE^2} + \frac{i\mathbf{p}\lambda_p}{2p} + \frac{iH_u \mathbf{p}}{2E^2} + \frac{\rho_3 \mathbf{c} H_u}{p} + \overset{(+)}{a}_\alpha \rho_1 \rho_3 p B_\alpha \mathbf{c} + i\overset{(-)}{a}_\alpha \rho_1 C_\alpha (\mathbf{c} \times \mathbf{p}) + \overset{(+)}{d}_\alpha \rho_3 p C_\alpha \mathbf{c} - i\overset{(-)}{d}_\alpha B_\alpha (\mathbf{c} \times \mathbf{p}) \tag{2.11}$$

Here, and in the following summation over the repeated index is understood.

For an integer spin s (Mathews, 1966a)

$$B_0 = \prod_{\mu=1}^{s} \frac{\lambda_p^2 - \mu^2}{-\mu^2}, \qquad B_\alpha = \frac{\lambda_p^2}{\alpha^2} \prod_{\mu\delta}^{s}{}' \frac{\lambda_p^2 - \mu^2}{\alpha^2 - \mu^2}; \qquad C_\alpha = \frac{\lambda_p}{\alpha} \prod_{\mu\delta}^{s}{}' \frac{\lambda_p^2 - \mu^2}{\alpha^2 - \mu^2} \tag{2.12}$$

For a half-odd integer spin s,

$$B_\alpha = \prod_{\mu=\frac{1}{2}}^{s}{}' \frac{\lambda_p^2 - \mu^2}{\alpha^2 - \mu^2}, \qquad C_\alpha = \frac{\lambda_p}{\alpha} \prod_{\mu=\frac{1}{2}}^{s}{}' \frac{\lambda_p^2 - \mu^2}{\alpha^2 - \mu^2} \tag{2.13}$$

$$[\alpha, \mu = s(s-1), (s-2), \ldots, 0 \text{ or } \tfrac{1}{2}]$$

For spins $\frac{1}{2}$ and 1, (2.11) reduces respectively to

$$\mathbf{K}_u = t\mathbf{p} - \mathbf{x}H_u + i\boldsymbol{\lambda} \tag{2.14}$$

$$\mathbf{K}_u = t\mathbf{p} - \mathbf{x}H_u + \frac{i\mathbf{p}\lambda_p(\rho_1 H_u)^2}{2pE^2} + \frac{i[\lambda_p E^2 p + p^2 H_u]\mathbf{p}}{2p^2E^2} + \frac{\rho_3 \boldsymbol{\tau} H_u}{p} + \frac{m[i\lambda_p{}^2\boldsymbol{\lambda} - i\boldsymbol{\lambda} - \rho_3\lambda_p \boldsymbol{\tau} - E\rho_1\rho_3\boldsymbol{\tau}/p]}{\sqrt{(p^2 + E^2)}} \tag{2.15}$$

For the operator $\mathbf{K}_u$ to be Hermitian, we require $\mathbf{K}_u^\dagger = \mathbf{K}_u$, which in turn leads to the condition

$$\mathbf{x}H_u - H_u\mathbf{x} = \frac{i\mathbf{p}}{2pE^2}\{\lambda_p(\rho_1 H_u)^2 + (H_u\rho_1)^2\lambda_p\} + i\lambda_p\mathbf{p}/p$$
$$+ \{\rho_3\boldsymbol{\tau}H_u - H_u\boldsymbol{\tau}\rho_3\}/p + i\mathbf{p}H_u/E^2 \qquad (2.16)$$

Using the commutation relations developed by Seetharaman *et al.* (1971), we get

$$\begin{aligned}\mathbf{x}H_u - H_u\mathbf{x}\\ &= i(b_{\alpha+1} - b_{\alpha-1})\rho_1 C_\alpha\boldsymbol{\lambda}/2p - i(b_{\alpha+1} + b_{\alpha-1} - 2b_\alpha)\rho_1 B_\alpha i\rho_3\boldsymbol{\tau}/2p\\ &\quad - i\left\{\alpha(b_{\alpha+1} - b_{\alpha-1}) - 2p\frac{db_\alpha}{dp}\right\}\rho_1 B_\alpha\mathbf{p}/2p^2 + i(k_{\alpha+1} - k_{\alpha-1})B_\alpha\boldsymbol{\lambda}/2p\\ &\quad - i(k_{\alpha+1} + k_{\alpha-1} - 2k_\alpha)C_\alpha i\rho_3\boldsymbol{\tau}/2p - \left\{\alpha(k_{\alpha+1} - k_{\alpha-1}) - 2p\frac{dk_\alpha}{dp}\right\}\\ &\quad \times C_\alpha\mathbf{p}/2p^2 \end{aligned} \qquad (2.17)$$

Coming to the right-hand side of equation (2.16), we have

$$\frac{i\mathbf{p}[\lambda_p(\rho_1 H_u)^2 + (H_u\rho_1)^2\lambda_p]}{2pE^2} + i\lambda_p\mathbf{p}/p$$
$$= -i2\rho_1\alpha k_\alpha b_\alpha B_\alpha\mathbf{p}/pE^2 + 2i\alpha b_\alpha^2 C_\alpha\mathbf{p}/pE^2 \qquad (2.18)$$

$$\begin{aligned}(\rho_3\boldsymbol{\tau}H_u - H_u\boldsymbol{\tau}\rho_3)/p\\ &= \rho_1\rho_3(b_{\alpha+1} + b_{\alpha-1} - 2b_\alpha)B_\alpha\boldsymbol{\tau}/2p - i\rho_1\alpha(b_{\alpha+1} - b_{\alpha-1})B_\alpha p/2p^2\\ &\quad + i\rho_1(b_{\alpha+1} - b_{\alpha-1})C_\alpha\boldsymbol{\lambda}/2p + i(k_{\alpha+1} - k_{\alpha-1})B_\alpha\boldsymbol{\lambda}/2p\\ &\quad - (2k_\alpha - k_{\alpha+1} - k_{\alpha-1})C_\alpha\rho_3\boldsymbol{\tau}/2p - i\alpha(k_{\alpha+1} - k_{\alpha-1})C_\alpha\mathbf{p}/2p^2\\ &\quad k_\alpha = E\tanh 2\alpha\omega; \qquad b_\alpha = E\,\mathrm{sech}\,2\alpha\omega \end{aligned} \qquad (2.19)$$

By using (2.18), (2.19) and (2.17), one can easily verify the equality of both sides of relation (2.16) and that ensures the Hermitian property of the operator‡ K_u.

3. *Observables in the U-Representation*

In the rest frame, where the particle is described by the wave function ψ_R, the Lorentz invariant inner product is defined as $(\psi_R, \psi_R) = \int \psi_R^\dagger\psi_R d^3x$.

‡ The author expresses his sincere thanks to Mr. K. Mailsamy and Mr. V. Krishnaswamy for their kind assistance in evaluating the expression for the boosts operator $\mathbf{K}_u$.

With respect to this inner product, the expectation value of the position or the spin of the particle is defined as

$$\left\langle \begin{matrix} \text{position} \\ \text{or} \\ \text{spin} \end{matrix} \right\rangle = \int \psi_R^\dagger \begin{pmatrix} \mathbf{x} \\ \text{or} \\ \mathbf{S} \end{pmatrix} \psi_R \, d^3x \tag{3.1}$$

where $\mathbf{x}$ and $\mathbf{S}$ are the position and spin operators in the rest frame. Similar expressions can be obtained in the U-representation by replacing ψ_R by $F\psi_u$. Accordingly, we have

$$(\psi_u, \psi_u) = \int \psi_u^\dagger F^\dagger F \psi_u \, d^3x = \int \psi_u^\dagger \psi_u \, d^3x \tag{3.2}$$

$$\left\langle \begin{matrix} \text{position} \\ \text{or} \\ \text{spin} \end{matrix} \right\rangle = \int \psi_u^\dagger F^\dagger \begin{pmatrix} \mathbf{x} \\ \text{or} \\ \mathbf{S} \end{pmatrix} F \psi_u \, d^3x \tag{3.3}$$

This shows that in the U-representation, the position operator $\mathbf{X}_u$ and the spin operator $\mathbf{S}_u$ are, respectively, equal to $F^\dagger \mathbf{x} F$ and $F^\dagger \mathbf{S} F$. By virtue of relation (2.2) we obtain

$$\begin{aligned} \mathbf{X}_u = F^\dagger \mathbf{x} F = \mathbf{x} - i\mathbf{p}\frac{\lambda_p \rho_1 [H_u \rho_1 + \rho_1 H_u]}{2pE^2} - \frac{\rho_3 \boldsymbol{\tau}}{p} \\ - [\overset{(+)}{a}_\alpha \rho_1 \rho_3 B_\alpha \boldsymbol{\lambda} + i\overset{(-)}{a}_\alpha \rho_1 C_\alpha \boldsymbol{\tau}] \times \mathbf{p} H_u/mE \\ + [i\overset{(-)}{d}_\alpha B_\alpha \boldsymbol{\tau} - \overset{(+)}{d}_\alpha \rho_3 C_\alpha \boldsymbol{\lambda}] \times \mathbf{p} H_u/mE \end{aligned} \tag{3.4}$$

$$\begin{aligned} \mathbf{S}_u = F^{-1} \mathbf{S} F = \frac{(\mathbf{S}\cdot\mathbf{p})\mathbf{p}}{p^2} + [\overset{(+)}{a}_\alpha \rho_1 \rho_3 B_\alpha \boldsymbol{\lambda} + i\overset{(-)}{a}_\alpha \rho_1 C_\alpha \boldsymbol{\tau} \\ + \overset{(+)}{d}_\alpha \rho_3 C_\alpha \boldsymbol{\lambda} - i\overset{(-)}{d}_\alpha B_\alpha \boldsymbol{\tau}] \times \mathbf{p} H_u/mE \end{aligned} \tag{3.5}$$

For spins $\frac{1}{2}$ and 1, equation (3.4) reduces to

$$\mathbf{X}_u = \mathbf{x} - \frac{i\mathbf{p}\lambda_p \rho_1 [\rho_1 H_u + H_u \rho_1]}{2pE^2} - \frac{\rho_3 \boldsymbol{\tau}}{p} + \frac{\rho_1 \rho_3 \boldsymbol{\tau} H_u}{pE} \tag{3.6}$$

$$\begin{aligned} \mathbf{X}_u = \mathbf{x} - \frac{i\mathbf{p}\lambda_p \rho_1 [\rho_1 H_u + H_u \rho_1]}{2pE^2} - \frac{\rho_3 \boldsymbol{\tau}}{p} \\ + [i\boldsymbol{\lambda} - i\lambda_p{}^2 \boldsymbol{\lambda} + \rho_3 \lambda_p \boldsymbol{\tau} + \rho_1 \rho_3 E \boldsymbol{\tau}/p] H_u \{E\sqrt{p^2 + E^2})\}^{-1} \end{aligned} \tag{3.7}$$

and equation (3.5) reduces to‡

‡ The relations (3.8) and (3.6) agree with the corresponding ones given in Mathews (1966b).

$$\mathbf{S}_u = \frac{(\mathbf{S}\,.\,\mathbf{p})\mathbf{p}}{p^2} - \frac{\rho_1\rho_3(\boldsymbol{\tau} \times \mathbf{p})H_u}{pE} \tag{3.8}$$

$$\mathbf{S}_u = \frac{(\mathbf{S}\,.\,\mathbf{p})\mathbf{p}}{p^2} + \{i\lambda_p{}^2\boldsymbol{\lambda} - i\boldsymbol{\lambda} - \rho_3\rho_3\lambda_p\boldsymbol{\tau} - \rho_1\rho_3E\boldsymbol{\tau}/p\} \times \frac{\mathbf{p}H_u}{E\sqrt{(p^2+E^2)}} \tag{3.9}$$

The invariance of the inner product given in (3.2) can be established as follows. After an infinitesimal Lorentz transformation, the inner product given in (3.2) takes the form,

$$(\psi_u', \psi_u') = \int \psi_u'^{\dagger}\psi_u'\, d^3x \tag{3.10}$$

where

the transformed wave function ψ_u' is related to ψ_u by the relation

$$\psi_u' = (I + i\mathbf{K}_u\,.\,\mathbf{dv})\psi_u \tag{3.11}$$

where $\mathbf{dv}$ is the parameter characterising the infinitesimal Lorentz transformation. From (3.10) and (3.11) we have:

$$(\psi_u', \psi_u') = \int \psi_u^{\dagger}(I - i\mathbf{K}_u^{\dagger}\,.\,\mathbf{dv})(I + i\mathbf{K}_u\,.\,\mathbf{dv})\psi_u\, d^3x$$

$$= \int \psi_u^{\dagger}\psi_u\, d^3x = (\psi_u, \psi_u)$$

since $\mathbf{K}_u^{\dagger} = \mathbf{K}_u$.

4. *The Extreme Relativistic Limit of the U-Representation*

A wave equation, appropriate for the description of particles with extreme relativistic velocities (the E-representation), was first obtained by Cini and Touschek (1958) by suitably projecting the Dirac representation to such relativistic limits. The method given by them was recently generalised for arbitrary spin (Alagar Ramanujam, 1973). The E-representation given in this section, unlike the one given by Alagar Ramanujam (1973), remains unitary for all spins.

The rest frame wave function ψ_R and the E-representation wave function ψ_u^E are related by the form (Alagar Ramanujam, 1974).

$$\psi_R = G\psi_u{}^E$$

$$G = \left[\frac{1}{\sqrt{2}}\right](I + \rho_1\Sigma C_\alpha) \tag{4.1}$$

where I is a $2(2s + 1)$ dimensional unit operator. The wave function $\psi_u{}^E$ and ψ are linked by the relation,

$$\psi = A'\psi_u{}^E \tag{4.2}$$

where

$$A' = R^{-1}G$$

The Hamiltonian $(H_u{}^E)$ of this representation takes the form (Alagar Ramanujam, 1974)

$$H_u{}^E = E\Sigma C_\alpha \tag{4.3}$$

Now, one can repeat the procedure given in the Sections 2 and 3 and obtain the following expressions for the boosts operator $\mathbf{K}_u{}^E$, the inner product $(\psi_u{}^E, \psi_u{}^E)$, position operator $\mathbf{X}_u{}^E$ and the spin operator $\mathbf{S}_u{}^E$, relevant for this representation.

$$\mathbf{K}_u{}^E = t\mathbf{p} - \mathbf{x}H_u{}^E - (m\rho_1 + mC_{\nu_0})[\rho_3 B_{\nu_0}\boldsymbol{\tau} + i\nu_0 B_{\nu_0}\mathbf{p}/p - iC_{\nu_0}\boldsymbol{\lambda}] + \rho_3 p\boldsymbol{\tau}H_u{}^E/E(E+m) + 2\mathbf{p}H_u{}^E/2E^2; \qquad \nu_0 = 0 \text{ or } \tfrac{1}{2} \tag{4.4}$$

$$(\psi_u{}^E, \psi_u{}^E) = \int \psi_u^{E\dagger}\psi_u{}^E\, d^3x \tag{4.5}$$

$$\mathbf{X}_u{}^E = \mathbf{x} - (\rho_1 + C_{\nu_0})[\rho_3 C_{\nu_0}\boldsymbol{\lambda} + iB_{\nu_0}\boldsymbol{\tau}] \times \mathbf{p}/2p^2 \tag{4.6}$$

$$\mathbf{S}_u{}^E = \mathbf{S} + (\rho_3\rho_1 - \rho_3 C_{\nu_0})[C_{\nu_0}\boldsymbol{\lambda} + i\rho_3 B_{\nu_0}\mathbf{C} - \nu_0 B_{\nu_0}\mathbf{p}/p]/2 \tag{4.7}$$

For spin $\frac{1}{2}$, equations (4.4), (4.6) and (4.7) reduce to

$$\mathbf{K}_u{}^E = t\mathbf{p} - \mathbf{x}H_u{}^E + i(E/p)\boldsymbol{\lambda} - i(m^2/p^3E)(\boldsymbol{\lambda}\cdot\mathbf{p})\mathbf{p} - \frac{m\rho_1}{p^2}(\mathbf{S}\times\mathbf{p}) \tag{4.8}$$

$$\mathbf{X}_u{}^E = \mathbf{x} + i\rho_1\boldsymbol{\lambda}/p - i\rho_1\lambda_p\mathbf{p}/p^2 - \rho_3\boldsymbol{\tau}/p \tag{4.9}$$

$$\mathbf{S}_u{}^E = (\mathbf{S}\cdot\mathbf{p})\mathbf{p}/p^2 - i\rho_1\boldsymbol{\tau} \tag{4.10}$$

The relations (4.8), (4.9) and (4.10) agree with the corresponding ones given in Alagar Ramanujam (1973).

5. *Discussion*

Now that all the operators relevant to the unitary representation have been determined, the next step will be to consider its second quantisation. The quantisation technique developed by Mathews (1967) can be simply borrowed and applied here. Preliminary work in this direction shows that the Hamiltonian (H_u) could lead to a satisfactory quantisation theory consistent with the microcausability condition only in the case of half-odd integer spins, but not in the case of integer spins. This means that the problem of determining a suitable Hamiltonian in the unitary representation which could be quantised consistently with the microcausality condition for integer spins, is to be investigated. The work related to this particular matter will be reported in a future publication.

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